

# Analog curved spacetimes in the reversed dissipation regime of cavity optomechanics

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In this paper, we theoretically propose an optomechanical scheme to explore the possibility of simulating the propagation of the collective excitations of the photon fluid in a curved spacetime. For this purpose, we introduce two theoretical models for two-dimensional photon gas in a planar optomechanical microcavity and a two-dimensional array of coupled optomechanical systems. In the reversed dissipation regime (RDR) of cavity optomechanics where the mechanical oscillator reaches equilibrium with its thermal reservoir much faster than the cavity modes, the mechanical degrees of freedom can adiabatically be eliminated. The adiabatic elimination of the mechanical mode provides an effective nonlinear Kerr-type photon-photon interaction. Using the nonlinear Schrödinger equation (NLSE), we show that the phase fluctuations in the two-dimensional photon fluid obey the Klein-Gordon equation for a massless scalar field propagating in a curved spacetime. The results reveal that the photon fluid as well as the corresponding metric can be controlled by manipulating the system parameters.

## I. INTRODUCTION

Investigation of an uncontrollable or inaccessible quantum system or quantum systems with a large number of degrees of freedom via some controllable quantum system is known as quantum simulation [1–3]. To this end, a wide variety of systems such as atoms in optical lattices, trapped ions, nuclear spins, superconducting circuits and photonic systems have been proposed as quantum simulators in different branches of physics such as condensed matter physics, high-energy physics, and cosmology [3].

The general theory of relativity (GTR) is, without doubt, one of the fascinating theories of the history of science describing the most important gravitational phenomena. The light deflection in gravitational field, gravitational waves, and also appearance of black holes, are some famous and spectacular predictions of GTR. One of the most important goals of recent investigations in theoretical physics is the presentation of a unified theory for gravitation and other quantum fields by their quantization on a curved background, i.e., a quantum field theory (QFT) in curved spacetime. This theory has some experimental consequences, such as particle creation, Hawking radiation, etc. (see e.g., [4] and references therein). Even in the case of small curvature, various aspects of QFT in curved spacetime cannot be examined directly with current technologies. Nevertheless, they can be investigated via quantum simulation. The geometric formulation of quantum mechanics is a very important step toward the presentation of the analog models of GTR in quantum realm. Hence, the investigation of novel models and a better understanding of previous analog models is still open-ended [5–9]. Various analog models of GTR have been proposed, such as the acoustic fluid [10], liquid helium [11, 12], Fermi gases [13, 14], slow light [15–18], nonlinear electromagnetic waveguides [19], graphene [20, 21], ion rings [22] and Bose-Einstein condensate (BEC) [23–29].

Many of the notions of atomic and molecular physics can

be realized on an entirely different scale, namely, in optomechanical systems. The field of optomechanics is currently undergoing rapid experimental and theoretical progress. Typically, electromagnetic radiation and macroscopic mechanical oscillators can interact via radiation pressure in a Fabry-Perot cavity with a movable end mirror. This interaction is the basis of various optomechanical phenomena which can occur in a wide range of system sizes and parameters (for review, see [30–33]). Many of the rudiments of quantum optomechanics date back to the early attempts of the investigation of one of the most prominent predictions of GTR, i.e., the gravitational waves. However, during the past decade, significant efforts have been devoted to developing and implementing optomechanical interaction for position or force sensing [34–37], backaction cooling [38, 54], quantum state transfer [40], optomechanical entanglement generation [41–43], optomechanically induced transparency realization [44–46], and generating self-sustained mechanical oscillations [47]. Another interesting aspect of the optomechanical systems, which has not been investigated much yet, is that they can be considered as promising candidate systems for quantum simulation applications. For instance, it has very recently been shown [48] how to realize the so-called sphere-coherent motional states of a mechanical oscillator in an optomechanical cavity in the presence of a two-level atom and how to use the optomechanical parameters to control the curvature of the sphere.

In the present paper, we investigate the possibility of simulating the curved spacetimes by photonic fields interacting with a mechanical oscillator operating in the reversed dissipation regime (RDR) of optomechanics [49, 50]. Due to the radiation pressure force, an effective Kerr-type photon-photon interaction arises which is mediated by the mirror motion. The emergence of fluid-like behavior for photons is the consequence of this photon-photon interaction so that we can use the hydrodynamic equations for the photonic fluid to study the corresponding effective metric for the propagation of fluctuations in the photon fluid. In this manner, the optomechanical system can be regarded as a quantum simulator for the propagation of the collective excitations of photon fluid in a curved spacetime. We also show that the photonic fluid as well as the corresponding curved spacetime can be controlled by manip-

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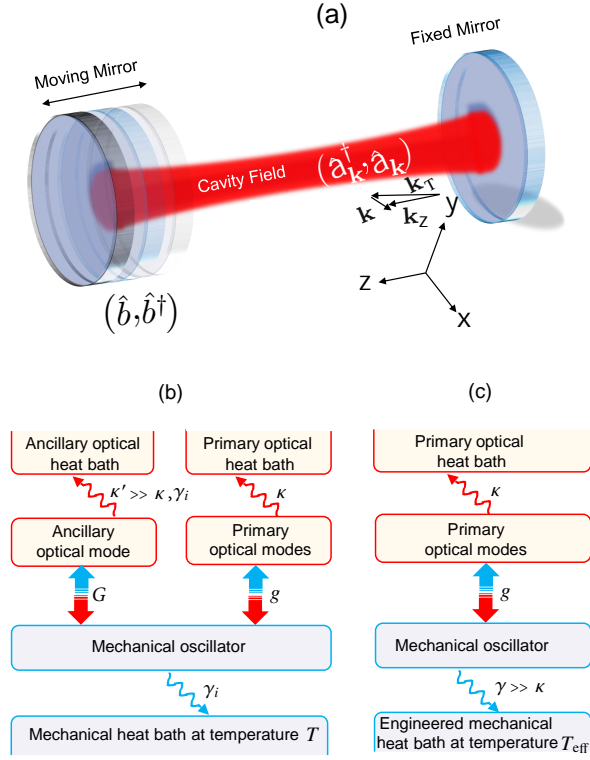


FIG. 1. (Color online). Schematic illustration of a Fabry-Perot cavity with a movable end mirror as the mechanical element interacting with a continuum of optical modes. (b) The ancillary optical field with a high damping rate ( $\kappa' \gg \kappa, \gamma_i$ ) is used to reach the RDR of cavity optomechanics. (c) RDR regime of optomechanics where  $\gamma \gg \kappa$  (see appendix A for details of calculations).

ulating the system parameters.

The rest of the paper is structured as follows. In Section II, we introduce the quantum field description of our first proposed system, i.e., a planar optomechanical microcavity. We discuss the equations of motions in the RDR of optomechanics. In Section III, we present our second proposed setup which is a two-dimensional optomechanical array. In Section IV, we discuss the so-called hydrodynamic version of the non-linear Schrödinger equation (NLSE), which can be linearized to describe an effective metric for the two-dimensional photon fluid. Finally, in Section V, we present our concluding remarks as well as some possible outlooks of our work..

## II. QUANTUM FIELD DESCRIPTION OF A PLANAR OPTOMECHANICAL MICROCAVITY

As the first theoretical proposal, we consider a planar optomechanical microcavity. As depicted in Fig (1-a), we consider a typical optomechanical system i.e., a Fabry-Pérot cavity with a movable end mirror, where the displacement of the mirror shifts the cavity resonances and results in the dependence of cavity resonance frequency to the position of the mechanical oscillator. We separate the total wave vector

$\mathbf{k}_T = (k_x, k_y, k_z)$ , into the transversal in-plane,  $\mathbf{k} = k_x \hat{x} + k_y \hat{y}$ , and the longitudinal,  $k_z$ , components. The photon energy dispersion in the limit of paraxial approximation as a function of in-plane and longitudinal wave vectors reads,

$$\hbar\omega_c(\mathbf{k}, k_z) = \hbar c \sqrt{\mathbf{k}^2 + k_z^2}, \quad (1)$$

where  $c$  denotes the speed of light. Imposing the boundary conditions on the electromagnetic field inside the optomechanical cavity with curved mirrors of radius  $R$  leads to a discrete wave vector in the  $z$  direction, namely,  $k_z(r) = n\pi/[l(r) + z]$ , where the mirrors separation at the distance  $r$  from the cavity axis is given by  $l(r) = l_0 - 2(R - \sqrt{R^2 - r^2})$  and the displacement of the mirror from its equilibrium position is indicated by  $z$ . In the limit of paraxial approximation ( $r \ll R, |\mathbf{k}| \ll k_z$ ) the photon dispersion relation becomes

$$\begin{aligned} \hbar\omega_c(\mathbf{k}, r) &= \hbar c \sqrt{\mathbf{k}^2 + \left(\frac{n\pi}{l(r) + z}\right)^2} \\ &\simeq mc^2 + \frac{\hbar^2 \mathbf{k}^2}{2m} + V_t(r) - \hbar g_0 z, \end{aligned} \quad (2)$$

where  $V_t(r) = m\Omega^2 r^2/2$  is the trapping potential and  $g_0 = n\pi c/l_0^2$  is the single-photon optomechanical coupling rate. Therefore, in a Fabry-Pérot cavity with a movable end mirror, spatial confinement of the photons in the  $z$  direction breaks the symmetry and eventually it provides a finite effective mass ( $m = \hbar n\pi/c l_0$ ) for the photons in the plane perpendicular to the cavity axis [51]. This makes the photon gas in the  $xy$ -plane to be effectively two-dimensional. Moreover, the mirror curvature provides a trapping potential with frequency  $\Omega = c \sqrt{2/l_0 R}$ . It should be noted that in the case  $z = 0$  (cavity with fixed mirrors) we recover the results of Ref. [51].

We now consider the case in which the moving mirror is interacting with a continuum of optical modes, schematically shown in Fig. (1-a). The moving mirror is treated as a single-mode quantum mechanical harmonic oscillator with effective mass  $m_m$ , frequency  $\omega_m$ , and the position operator  $\hat{z} = z_0(\hat{b} + \hat{b}^\dagger)$  where  $\hat{b}$  and  $\hat{b}^\dagger$  satisfy the commutation relation  $[\hat{b}, \hat{b}^\dagger] = 1$  and  $z_0 = \sqrt{\hbar/2m_m\omega_m}$  is the zero-point fluctuation of the mirror's position. The single-mode consideration is valid if the detection bandwidth is chosen such that it includes only a single, isolated, mechanical resonance and mode-mode coupling is negligible [52]. The continuum of optical modes, characterized by transverse wave vector  $\mathbf{k}$  and the annihilation (creation) operator  $a_{\mathbf{k}}$  ( $a_{\mathbf{k}}^\dagger$ ), is coupled to the movable mirror via the radiation pressure coupling. The optical field operators satisfy the commutation relations  $[\hat{a}_{\mathbf{k}}, \hat{a}_{\mathbf{k}'}^\dagger] = (2\pi)^2 \delta(\mathbf{k} - \mathbf{k}')$  and  $[\hat{a}_{\mathbf{k}}, \hat{a}_{\mathbf{k}'}] = 0 = [\hat{a}_{\mathbf{k}}^\dagger, \hat{a}_{\mathbf{k}'}^\dagger]$ . In other words, the cavity free spectral range is much larger than the mechanical frequency (the single-longitudinal-mode assumption). We restrict our considerations for the primary mode to the case of a single-longitudinal-cavity mode. Using the dispersion relation of Eq. (2), the total Hamiltonian of the system is expressed as

$$\hat{\mathcal{H}}/\hbar = \int \frac{d^2 \mathbf{k}}{(2\pi)^2} [\Delta_{\mathbf{k}} + V_t(r) - g(\hat{b} + \hat{b}^\dagger)] \hat{a}_{\mathbf{k}}^\dagger \hat{a}_{\mathbf{k}} + \omega_m \hat{b}^\dagger \hat{b}, \quad (3)$$

where we have defined  $\Delta_{\mathbf{k}} = mc^2/\hbar + \hbar\mathbf{k}^2/2m$ . This Hamiltonian consists of the free energy of the single mechanical mode and the optical modes, the trapping potential due to the mirror curvature, and the optomechanical interaction energy. We have also defined  $g = g_0 z_0$ , with  $g_0$  being the single-photon optomechanical strength, as the effective optomechanical coupling strength.

We investigate the dynamics of the system by the Heisenberg-Langevin equations associated with the Hamiltonian of Eq. (3)

$$\partial_t \hat{a}_{\mathbf{k}} = -i \left[ \Delta_{\mathbf{k}} + \frac{1}{2\hbar} m \Omega^2 r^2 - g(\hat{b} + \hat{b}^\dagger) \right] \hat{a}_{\mathbf{k}}, \quad (4)$$

$$\partial_t \hat{b} = -i(\omega_m - i\gamma/2) \hat{b} + ig \int \frac{d^2\mathbf{k}}{(2\pi)^2} \hat{a}_{\mathbf{k}}^\dagger \hat{a}_{\mathbf{k}} + \sqrt{\gamma} \hat{b}^{in}(t), \quad (5)$$

where the zero-mean operator  $\hat{b}^{in}(t)$  denotes the mechanical noise operator. Here, we assume that the fluctuation-dissipation process affects only the mechanical mode. This assumption is justified by using a cavity with a high-quality factor (small damping rate  $\kappa$ ) and a mechanical mode with a large damping rate. Of course, it is not the normal dissipation regime of optomechanics where the mechanical dissipation rate  $\gamma$  is much smaller than the cavity linewidth  $\kappa$ . Therefore, one has to engineer a mechanical mode with an effective large damping rate. The mechanism of realizing RDR of optomechanics is depicted in Figs. (1-b) and (1-c). The detailed considerations for the realization of the RDR in the system under study are described in Appendix A. Briefly, RDR with respect to the optical mode  $\hat{a}_{\mathbf{k}}$  can be achieved by optomechanical sideband cooling of a high-Q mechanical mode with an ancillary optical mode possessing a large damping rate  $\kappa' \gg \kappa$ . In principle, for time scales longer than  $\gamma^{-1}$  and much shorter than  $\kappa^{-1}$  we conclude that the photonic subsystem becomes isolated from the environment. Based on the calculations in Appendix A, for a typical optomechanical system with a mirror with intrinsic frequency  $\omega_i = 2\pi \times 10\text{MHz}$  interacting with an ancillary optical mode with damping rate  $\kappa' = 1.2\text{MHz}$ , it is possible to increase the mechanical damping rate to  $\gamma = 10\text{GHz}$ .

Since the mechanical damping rate is much greater than the cavity decay rate ( $\gamma \gg \kappa$ ), the mechanical mode can be adiabatically eliminated on time scales greater than  $\gamma^{-1}$ . For this purpose, one can formally integrate Eq. (5) to obtain

$$\begin{aligned} \hat{b}(t) = & \hat{b}(0) e^{-i(\omega_m - i\gamma/2)t} + ig \int_0^t \frac{d^2\mathbf{k}}{(2\pi)^2} dt' n_{\mathbf{k}}(t') e^{i(\omega_m - i\gamma/2)(t'-t)} \\ & + \sqrt{\gamma} \int_0^t dt' e^{i(\omega_m - i\gamma/2)(t'-t)} \hat{b}^{in}(t'). \end{aligned} \quad (6)$$

For time scales long compared to  $\gamma^{-1}$ , the first term becomes zero. Substituting this equation into Eq. (4) the following

equation is obtained for the optical mode

$$\begin{aligned} \partial_t \hat{a}_{\mathbf{k}} = & -i \left[ \Delta_{\mathbf{k}} + \frac{1}{2\hbar} m \Omega^2 r^2 \right] \hat{a}_{\mathbf{k}} + \hat{f}_{\mathbf{k}}^{in}(t) \\ & - 2ig^2 \left[ \int_0^t dt' e^{\gamma(t'-t)/2} \sin \omega_m(t'-t) \int \frac{d^2\mathbf{k}}{(2\pi)^2} \hat{n}_{\mathbf{k}}(t') \right] \hat{a}_{\mathbf{k}}, \end{aligned} \quad (7)$$

where we have introduced the generalized noise operator

$$\hat{f}_{\mathbf{k}}^{in}(t) = \sqrt{\gamma} g \hat{a}_{\mathbf{k}} \int_0^t dt' \left[ e^{i(\omega_m - i\gamma/2)(t'-t)} \hat{b}_{in}(t') + H.c. \right]. \quad (8)$$

For time scales much longer than the mechanical characteristic time  $\gamma^{-1}$  this generalized noise operator can be approximated by  $\hat{f}_{\mathbf{k}}^{in}(t) \simeq 0$ . The net effect of the mirror is a redistribution of the photons between the various transversal modes. Now, in the RDR the mirror motion can be adiabatically eliminated, resulting in a mechanical field that is affected too much by the optical field operators. Using this approximation and defining

$$\mathcal{T}(t) = - \int_0^t dt' e^{-\gamma(t-t')/2} \sin \omega_m(t'-t), \quad (9)$$

which for time scales much longer than  $\gamma^{-1}$  can be approximated as

$$\mathcal{T} \equiv \mathcal{T}(t \gg \gamma^{-1}) \simeq \frac{\omega_m}{\gamma^2/4 + \omega_m^2}, \quad (10)$$

Eq. (7) becomes

$$\partial_t \hat{a}_{\mathbf{k}} = -i \left[ \Delta_{\mathbf{k}} + \frac{1}{2\hbar} m \Omega^2 r^2 - 2g^2 \mathcal{T} \int \frac{d^2\mathbf{k}}{(2\pi)^2} \hat{n}_{\mathbf{k}} \right] \hat{a}_{\mathbf{k}} \quad (11)$$

Altogether, the adiabatic elimination of the mirror motion has two consequences on the dynamics of the system. First, it provides a reservoir for the photons inside the cavity that can exchange excitations. Second, it gives rise to a nonlinear Kerr-type photon-photon interaction with a modified interaction rate,  $2g^2\mathcal{T}$ .

The Fourier transform of  $\hat{a}_{\mathbf{k}}$  ( $\hat{a}_{\mathbf{k}}^\dagger$ ) is defined to be the two-dimensional cavity photon field operator  $\hat{\Psi}(t, \mathbf{r})$  ( $\hat{\Psi}^\dagger(t, \mathbf{r})$ )

$$\hat{\Psi}(t, \mathbf{r}) = \int \frac{d^2\mathbf{k}}{(2\pi)^2} \hat{a}_{\mathbf{k}} e^{i\mathbf{k} \cdot \mathbf{r}}, \quad (12)$$

where the field operators satisfy the following equal-time commutation relations

$$[\hat{\Psi}(t, \mathbf{r}), \hat{\Psi}^\dagger(t, \mathbf{r}')] = \delta(\mathbf{r} - \mathbf{r}'), \quad (13)$$

$$[\hat{\Psi}(t, \mathbf{r}), \hat{\Psi}(t, \mathbf{r}')] = 0 = [\hat{\Psi}^\dagger(t, \mathbf{r}), \hat{\Psi}^\dagger(t, \mathbf{r}')]. \quad (14)$$

Using the Fourier transform the equation of motion (11) becomes

$$\begin{aligned} i\hbar \partial_t \hat{\Psi}(t, \mathbf{r}) = & \left[ -\frac{\hbar^2}{2m} \nabla^2 + mc^2 - \hbar\omega_L + \frac{1}{2} m \Omega^2 r^2 \right] \hat{\Psi}(t, \mathbf{r}) \\ & - 2\hbar g^2 \mathcal{T} \left[ \int d^2\mathbf{r}' \hat{\Psi}^\dagger(t, \mathbf{r}') \hat{\Psi}(t, \mathbf{r}) \right] \hat{\Psi}(t, \mathbf{r}). \end{aligned} \quad (15)$$

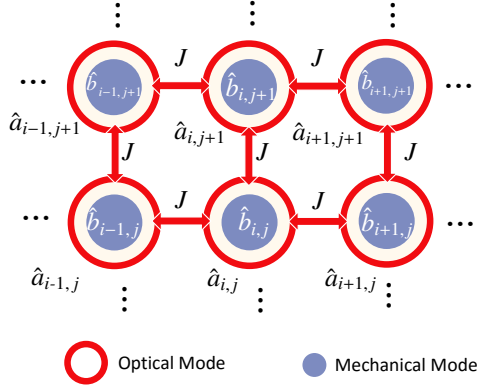


FIG. 2. (Color online). Schematic illustration of an array of a two-dimensional array of coupled optomechanical systems.

We now assume that the volume of integration is small and the integrand is slowly varying and smooth in the region of integration so that we can write approximately

$$\int d^2\mathbf{r} \hat{\Psi}^\dagger(t, \mathbf{r}) \hat{\Psi}(t, \mathbf{r}) \simeq \mathcal{V} \hat{\Psi}^\dagger(t, \mathbf{r}) \hat{\Psi}(t, \mathbf{r}), \quad (16)$$

where  $\mathcal{V}$  is the volume of the integration. In this manner, we arrive at a NLSE describing the two-dimensional photon field in the optomechanical microcavity

$$i\hbar \partial_t \hat{\Psi}(t, \mathbf{r}) = \left[ -\frac{\hbar^2}{2m} \nabla^2 + \tilde{V}_t(r) + \mathcal{G} |\hat{\Psi}(t, \mathbf{r})|^2 \right] \hat{\Psi}(t, \mathbf{r}), \quad (17)$$

where we have defined  $\tilde{V}_t(r) = mc^2 + V_t(r)$ , and the photon-photon coupling strength  $\mathcal{G}$

$$\mathcal{G} = -2\hbar g^2 \mathcal{T} \mathcal{V} = -2\hbar \omega_m \frac{g^2 \mathcal{V}}{\gamma^2 + \omega_m^2}, \quad (18)$$

is determined by the optomechanical parameters.

### III. ARRAY OF COUPLED OPTOMECHANICAL SYSTEMS

As the second theoretical proposal, we consider a two-dimensional array of coupled optomechanical systems schematically depicted in Fig (2). This system could be realized experimentally in optomechanical crystals. One- and two-dimensional optomechanical crystals have been recently realized experimentally [53, 54]. In this case, optical (mechanical) modes on each site interact locally via radiation pressure while photons can tunnel between adjacent site with hopping rate  $J$ . Since in such experiments phonon tunneling rate is much smaller (four orders of magnitude) than the photon tunneling rate, we can neglect phonon hopping in our considerations. The optomechanical array Hamiltonian is given

by [55–57]

$$\hat{H}/\hbar = \sum_{i,j} \left[ \omega_c \hat{a}_{ij}^\dagger \hat{a}_{ij} + \omega_m \hat{b}_{ij}^\dagger \hat{b}_{ij} - g \hat{a}_{ij}^\dagger \hat{a}_{ij} (\hat{b}_{ij}^\dagger + \hat{b}_{ij}) \right] + \sum_{\langle i,j,k \rangle} J (\hat{a}_{ij}^\dagger \hat{a}_{kj} + \hat{a}_{ij}^\dagger \hat{a}_{ik}), \quad (19)$$

where  $\hat{b}_{ij}$  and  $\hat{a}_{ij}$ , respectively, stands for the bosonic annihilation operators for phonon and photons on the lattice site denoted by  $(i, j)$ . In the last term  $\langle i, j, k \rangle$  denotes the summation over all adjacent lattice sites. Similar to the case of planar optomechanical microcavity, we consider the system in the RDR and write down the Heisenberg-Langevin equations for the quantum fields

$$\begin{aligned} \partial_t \hat{a}_{ij} = & -(i\omega_c + \kappa) \hat{a}_{ij} + ig(\hat{b}_{ij}^\dagger + \hat{b}_{ij}) \hat{a}_{ij} \\ & - iJ (\hat{a}_{i-1,j} + \hat{a}_{i+1,j} + \hat{a}_{i,j+1} + \hat{a}_{i,j-1}) \end{aligned} \quad (20)$$

$$\partial_t \hat{b}_{ij} = -(i\omega_m + \gamma) \hat{b}_{ij} + ig \hat{a}_{ij}^\dagger \hat{a}_{ij} - \sqrt{\gamma} \hat{b}_{ij}^{in}(t) \quad (21)$$

For a slowly varying optical field which is justified when the optical wave vector  $|\mathbf{k}| \ll 1/h$ , we can take the field operators to be continuous functions of  $x = ih$  and  $y = jh$  where  $h$  is the cells spacing, i.e.,  $\hat{a}_{ij}(t) \rightarrow \hat{\Psi}(t, \mathbf{r})$  and  $\hat{b}_{ij}(t) \rightarrow \hat{b}(t, \mathbf{r})$ . This assumption allows us to rewrite the equations of motion as follows

$$\begin{aligned} \partial_t \hat{\Psi}(t, \mathbf{r}) = & -(i\omega_c + \kappa) \hat{\Psi}(t, \mathbf{r}) + ig [\hat{b}^\dagger(t, \mathbf{r}) + \hat{b}(t, \mathbf{r})] \hat{\Psi}(t, \mathbf{r}) \\ & - iJ [\hat{\Psi}(t, \mathbf{r} - h\hat{x}) + \hat{\Psi}(t, \mathbf{r} + h\hat{x}) + \hat{\Psi}(t, \mathbf{r} - h\hat{y}) + \hat{\Psi}(t, \mathbf{r} + h\hat{y})] \end{aligned} \quad (22a)$$

$$\partial_t \hat{b}(t, \mathbf{r}) = -(i\omega_m + \gamma) \hat{b}(t, \mathbf{r}) + ig \hat{n}(t, \mathbf{r}) - \sqrt{\gamma} \hat{b}^{in}(t, \mathbf{r}) \quad (22b)$$

We now expand the field operators  $\hat{\Psi}(t, \mathbf{r} \pm h\hat{x})$  and  $\hat{\Psi}(t, \mathbf{r} \pm h\hat{y})$  as Taylor series up to second order in  $h$

$$\hat{\Psi}(t, \mathbf{r} \pm h\hat{x}) \simeq \hat{\Psi}(t, \mathbf{r}) \pm h \frac{\partial \hat{\Psi}(t, \mathbf{r})}{\partial x} + \frac{h^2}{2} \frac{\partial^2 \hat{\Psi}(t, \mathbf{r})}{\partial x^2} \quad (23)$$

$$\hat{\Psi}(t, \mathbf{r} \pm h\hat{y}) \simeq \hat{\Psi}(t, \mathbf{r}) \pm h \frac{\partial \hat{\Psi}(t, \mathbf{r})}{\partial y} + \frac{h^2}{2} \frac{\partial^2 \hat{\Psi}(t, \mathbf{r})}{\partial y^2} \quad (24)$$

Under this approximation, Eq. (22a) takes the form

$$\begin{aligned} \partial_t \hat{\Psi}(t, \mathbf{r}) = & -iJh^2 \nabla^2 \hat{\Psi}(t, \mathbf{r}) - (i\omega_c + \kappa + 4iJ) \hat{\Psi}(t, \mathbf{r}) \\ & + ig [\hat{b}^\dagger(t, \mathbf{r}) + \hat{b}(t, \mathbf{r})] \hat{\Psi}(t, \mathbf{r}) \end{aligned} \quad (25)$$

Again, applying the adiabatic approximation for the mechanical modes, we arrive at an equation analogous to Eq. (17) with  $m = \hbar/2Jh^2$ ,  $\mathcal{G} = -2\hbar g^2 \mathcal{T}$  and  $\tilde{V}_t = \hbar(\omega_c + 4J)$ . We should notice that in contrast to the former case, here photons gain mass due to the hopping between lattice sites. Moreover, the potential  $\tilde{V}_t$  could be generally space dependent by introducing a space dependent optical frequency  $\omega_c(\mathbf{r})$  or hopping rate  $J(\mathbf{r})$ .



#### IV. THE EFFECTIVE METRIC FOR THE PROPAGATION OF THE FLUCTUATIONS IN THE PHOTONIC FLUID

Using the mean-field approximation, the field operator can be written as the sum of its mean value and a fluctuation operator with zero mean value, that is,  $\hat{\Psi} = \Psi_0(1 + \hat{\phi})$ . The classical mean field and the fluctuation operator satisfy, respectively

$$i\hbar\partial_t\Psi_0 = \left[ -\frac{\hbar^2}{2m}\nabla^2 + \tilde{V}_t(r) + \mathcal{G}|\Psi_0|^2 \right] \Psi_0, \quad (26a)$$

$$i\hbar\partial_t\hat{\phi} = -\left[ \frac{\hbar^2}{2m}\nabla^2 + \frac{\hbar^2}{m}\frac{\nabla\Psi_0}{\Psi_0}\nabla \right] \hat{\phi} + n\mathcal{G}(\hat{\phi} + \hat{\phi}^\dagger), \quad (26b)$$

where  $n \equiv |\Psi_0|^2$  is defined to be the two-dimensional photon density.

In order to interpret clearly Eqs. (26a) and (26b), we write the quantum field operator,  $\hat{\Psi}$ , in the so-called Madulung representation, namely

$$\begin{aligned} \Psi_0 &= \sqrt{n}e^{i\theta}, \\ \hat{\Psi} &= \sqrt{n + \delta\hat{n}} e^{i(\theta + \delta\hat{\theta})} \simeq \Psi_0(1 + \frac{\delta\hat{n}}{2n} + i\delta\hat{\theta}), \end{aligned} \quad (27)$$

where  $\delta\hat{n}$  and  $\delta\hat{\theta}$  denote, respectively, the amplitude of the photon number density fluctuation and the phase fluctuation. The equations of motion for the density fluctuation and the phase fluctuation read

$$\partial_t\delta\hat{n} = -\nabla(\mathbf{v}_0\delta\hat{n} + \frac{\hbar n}{m}\nabla\delta\hat{\theta}), \quad (28a)$$

$$\hbar\partial_t\delta\hat{\theta} = -\hbar\mathbf{v}_0\nabla\delta\hat{\theta} - \frac{mc_{\text{ex}}^2}{n}\delta\hat{n} + \frac{mc_{\text{ex}}^2}{4n}\xi^2\nabla[n\nabla(\frac{\delta\hat{n}}{n})], \quad (28b)$$

where  $\mathbf{v}_0 = \hbar\nabla\theta/m$  and  $c_{\text{ex}} = \sqrt{n\mathcal{G}/m}$  are the local velocity of fluid and the local speed of excitations, respectively. The so-called healing length is defined to be  $\xi \equiv 1/mc_{\text{ex}}$ . Within the hydrodynamic approximation, i.e., over the length scales much larger than  $\xi$  the last term in Eq.(28b) can be safely ignored and thus

$$\delta\hat{n} \simeq -\frac{\hbar n}{mc_{\text{ex}}^2} \left[ \mathbf{v}_0\nabla\delta\hat{\theta} + \partial_t\delta\hat{\theta} \right]. \quad (29)$$

Combining Eq. (29) and Eq. (28a) results in

$$-(\partial_t + \nabla\mathbf{v}_0)\frac{n}{mc_{\text{ex}}^2}(\partial_t + \mathbf{v}_0\nabla)\delta\hat{\theta} + \nabla\frac{n}{m}\nabla\delta\hat{\theta} = 0. \quad (30)$$

These fluctuations, within the hydrodynamic approximation, are analogous to the collective quantum field on a curved metric. In fact, the photon phase fluctuation obeys the covariant Klein-Gordon equation for a massless scalar field propagating in a curved spacetime

$$\square\delta\hat{\theta} = 0, \quad (31)$$

where the d'Alembertian operator depends on an effective metric,  $g^{\mu\nu}$ , and is given by

$$\square = \frac{1}{\sqrt{-g}}\partial_\mu(\sqrt{-g}g^{\mu\nu}\partial_\nu), \quad (32)$$

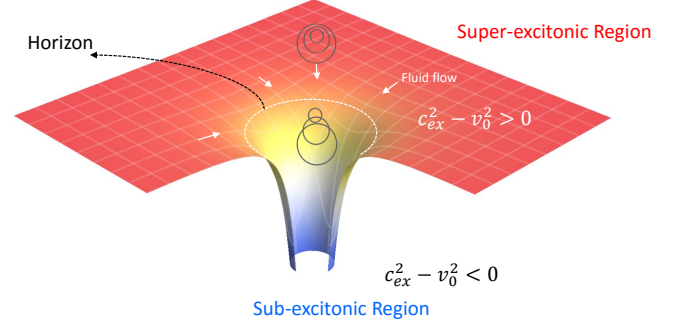


FIG. 3. (Color online). Schematic diagram of a curved analog spacetime with an analog black hole.

with

$$g_{\mu\nu} = \frac{n}{mc_s} \begin{bmatrix} -(c_{\text{ex}}^2 - \mathbf{v}_0 \cdot \mathbf{v}_0) & -v_0^i \\ -v_0^j & \delta_{ij} \end{bmatrix}. \quad (33)$$

Here, the Greek indices range from 0 to 2,  $(i, j)$  range from 1 to 2, and  $g$  is the determinant of  $g_{\mu\nu}$ . The line element of this spacetime is given by

$$ds^2 = g_{\mu\nu}dx^\mu dx^\nu = \frac{n}{mc_{\text{ex}}} \left[ -c_{\text{ex}}^2 dt^2 + (d\mathbf{r} - \mathbf{v}_0 dt)(d\mathbf{r} - \mathbf{v}_0 dt) \right]. \quad (34)$$

This derivation is similar to the derivation of analog spacetime given in [23–26, 29]. A singularity may occur if the flow velocity and the local excitation velocity have equal values. As depicted in Fig. (3), this configuration corresponds to a black hole since excitation waves traveling with  $c_{\text{ex}} < v_0$  are trapped inside the *superexcitonic* region and they are not able to propagate backward. Although analog models of GTR have a limited ability to simulate all aspects of GR, they provide accessible experimental models for quantum field theory in curved spacetime. Some properties (mainly kinematic) of GTR can be investigated by the analogies with accessible physical systems.

According to Eq. (A11) if  $\omega_{\text{opt}} > \omega_i$  then  $\mathcal{G}$  will be a positive parameter but it is physically unacceptable solution because the system enters an unstable region. Therefore, the parameter  $\mathcal{G}$  can only have a negative value for a bare optomechanical system. This is equivalent to the imaginary velocity for excitation waves and hence they feel a Euclidean metric. Positive nonlinearities can also be introduced into the system by placing an extra Kerr medium inside the optomechanical microcavity [58, 59]. This is equivalent to the real excitonic speed and, consequently, corresponds to a Lorentzian metric for the analog spacetime which is suitable for the investigation of analog black holes. In both cases, the velocity of cavity excitations can be controlled by the ancillary optical mode.

#### V. CONCLUSIONS

In summary, we have introduced a theoretical scheme for the quantum simulation of the curved spacetimes in two types

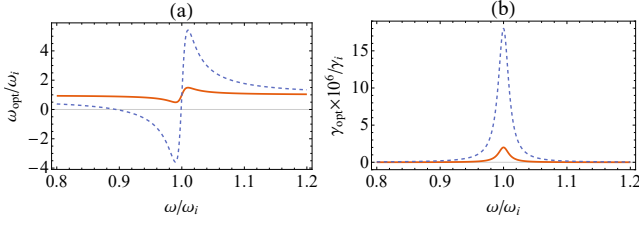


FIG. 4. (color online). (a) Modified frequency of the mechanical oscillator and (b) modified optomechanical damping rate versus the normalized frequency  $\omega/\omega_i$  for two different couplings of the ancillary mode:  $G = 0.1\omega_i$  (solid line),  $G = 0.3\omega_i$  (dashed line). Parameter values are  $\omega_i = 2\pi \times 10\text{MHz}$ ,  $\gamma_i/\omega_i = 10^{-5}$ ,  $\bar{\Delta} = -\omega_i$ ,  $\kappa' = 0.02\omega_i$ .

of optomechanical systems operating in the RDR, that is a planar optomechanical microcavity and a two-dimensional array of coupled optomechanical systems. Such optomechanical systems are realizable with the state-of-the-art technologies. Key features of our proposal can be categorized as follows. First, the optical mode with higher damping rate renormalizes both the damping rate and the frequency of the mechanical element (see Appendix A). Second, the mechanical oscillator which operates in the RDR of optomechanics can be adiabatically eliminated to achieve a Kerr-like photon-photon interaction. The description of the system with the NLSE allows us to connect the photonic fluid with the notion of the analog spacetime. We have shown that the phase fluctuation in the photonic fluid obeys the Klein-Gordon equation for a massless scalar field propagating in a curved spacetime with a metric given by Eq. (33) which can be regarded as an analog of the curved spacetime. The corresponding metric can be controlled by the system parameters.

The system introduced here is rather a simple system. As an outlook for future works, it can be extended to more complex situations which open up several possibilities. For example, by introducing two primary modes with different polarizations, it should be possible to produce a two-component photonic fluid inside an optical cavity. In principle, it could be possible to introduce an effective mass for the photons in two dimensions by adding an optical parametric amplifier in the cavity. Studying the quantum nature of the photon fluid through various quantum optical measurements on the leakage photon field is another outlook for future works.

#### Appendix A: Mechanical oscillator in the RDR

In this appendix, we discuss how to prepare the mechanical mode in the RDR of optomechanics. Assume that the mechanical mode with intrinsic frequency  $\omega_i$  is coupled to another ancillary optical mode denoted by  $\hat{a}$  via the radiation pressure. The Hamiltonian of the system in the frame rotating with the laser frequency  $\omega_L$  is given by

$$\hat{H}/\hbar = -\Delta \hat{a}^\dagger \hat{a} + \omega_i \hat{b}^\dagger \hat{b} + G_0 \hat{a}^\dagger \hat{a} (\hat{b}^\dagger + \hat{b}) + (\varepsilon^* \hat{a} + \varepsilon \hat{a}^\dagger), \quad (\text{A1})$$

where  $G_0$ ,  $\varepsilon$  and  $\Delta = \omega_L - \omega_c$  are the single-photon optomechanical coupling, laser pumping rate and the detun-

ing, respectively. Considering a strongly driven field and weak optomechanical coupling allows us to linearize the quantum dynamics of fluctuations around the semiclassical amplitudes,  $\hat{a} = \alpha + \hat{c}$  and  $\hat{b} = \beta + \hat{d}$ . The steady-state solutions for the classical mean fields are given by  $\alpha = -i\varepsilon/[-\Delta + G_0(\beta + \beta^*) - i\kappa']$  and  $\beta = -G_0|\alpha|^2/(\omega_i - i\gamma_i)$ . Here, the cavity damping rate for the ancillary mode is denoted by  $\kappa'$  and the intrinsic damping rate of the mechanical mode is denoted by  $\gamma_i$ . The linearized quantum Langevin equations for the fluctuations are obtained as

$$\partial_t \hat{c} = (i\bar{\Delta} - \kappa'/2)\hat{c} - iG(\hat{d} + \hat{d}^\dagger) + \sqrt{\kappa'}\hat{c}^{\text{in}}(t), \quad (\text{A2})$$

$$\partial_t \hat{d} = i(\omega_i - i\gamma_i/2)\hat{d} - i(G\hat{c}^\dagger + G^*\hat{c}) + \sqrt{\gamma_i}\hat{d}^{\text{in}}(t), \quad (\text{A3})$$

where we have defined  $\bar{\Delta} = \Delta + G_0(\beta + \beta^*)$  and  $G = G_0\alpha$ . The zero-mean operators  $\hat{c}^{\text{in}}(t)$  and  $\hat{d}^{\text{in}}(t)$  that denote, respectively, the vacuum optical input noise and the mechanical noise operator satisfy commutation relations  $[\hat{c}^{\text{in}}(t), \hat{c}^{\text{in},\dagger}(t')] = [\hat{d}^{\text{in}}(t), \hat{d}^{\text{in},\dagger}(t')] = \delta(t - t')$  and the second order correlations  $\langle \hat{d}^{\text{in},\dagger}(t)\hat{d}^{\text{in}}(t') \rangle = \bar{n}_{\text{th}}\delta(t - t')$ , and  $\langle \hat{c}^{\text{in}}(t)\hat{c}^{\text{in},\dagger}(t') \rangle = \delta(t - t')$  in which we have assumed that the cavity is at zero temperature and  $\bar{n}_{\text{th}} = (\exp(\hbar\omega_m/k_B T) - 1)^{-1}$  is the mean number of thermal phonons of the mechanical oscillator at heat bath temperature  $T$  with  $k_B$  being the Boltzmann constant. Equations (A2) and (A3) together with noise correlations fully describe the dynamics of the system under consideration. It is convenient to rewrite the equations of motion in the Fourier space

$$-i\omega\hat{c}[\omega] = (i\bar{\Delta} - \kappa'/2)\hat{c}[\omega] - iG(\hat{d}[\omega] + \hat{d}^\dagger[\omega]) - \sqrt{\kappa'}\hat{c}^{\text{in}}[\omega], \quad (\text{A4})$$

$$-i\omega\hat{d}[\omega] = -(i\omega_i + \gamma_i/2)\hat{d}[\omega] - i(G\hat{c}^\dagger[\omega] + G^*\hat{c}[\omega]) - \sqrt{\gamma_i}\hat{d}^{\text{in}}[\omega], \quad (\text{A5})$$

combining these two equations results in

$$\hat{d}[\omega] = \frac{-i\sqrt{\kappa'}\{G^*\chi[\omega]\hat{a}_{\text{in}}[\omega] + G\chi^*[-\omega]\hat{a}_{\text{in}}^\dagger[\omega]\} + \sqrt{\gamma_i}\hat{b}_{\text{in}}[\omega]}{i(\omega - \omega_i + \Sigma[\omega]) - \gamma_i/2}, \quad (\text{A6})$$

where  $\chi[\omega] = [-i(\omega + \bar{\Delta}) + \kappa'/2]^{-1}$  and  $\Sigma[\omega] = -i|G|^2(\chi[\omega] - \chi^*[-\omega])$  are the optical susceptibility and the self-energy, respectively. Therefore, the effect of the ancillary mode will be a renormalization in the mechanical damping rate ( $\gamma_i \rightarrow \gamma$ ) and the mechanical frequency ( $\omega_i \rightarrow \omega_m$ ). The finite cavity lifetime leads to the retarded essence of the radiation-pressure force and, consequently, it introduces the frequency-dependent mechanical frequency shift  $\omega_{\text{opt}}$  and optomechanical damping rate  $\gamma_{\text{opt}}$  given by [30–33, 60]

$$\gamma_{\text{opt}}(\omega) = \frac{|G|^2\omega_i}{\omega} \left[ \frac{\kappa'}{\kappa'^2/4 + (\bar{\Delta} + \omega)^2} - \frac{\kappa'}{\kappa'^2/4 + (\bar{\Delta} - \omega)^2} \right], \quad (\text{A7})$$

$$\omega_{\text{opt}}(\omega) = \frac{|G|^2\omega_i}{\omega} \left[ \frac{\bar{\Delta} + \omega}{\kappa'^2/4 + (\bar{\Delta} + \omega)^2} + \frac{\bar{\Delta} - \omega}{\kappa'^2/4 + (\bar{\Delta} - \omega)^2} \right]. \quad (\text{A8})$$

The intrinsic mechanical resonator damping rate and frequency modify due to the radiation pressure as follows

$$\gamma = \gamma_{\text{opt}} + \gamma_i, \quad \omega_m = \omega_{\text{opt}} + \omega_i. \quad (\text{A9})$$

One can thus increase or decrease the damping rate and frequency of the moving mirror, depending on the sign of the detuning. In particular, for a red-detuned ancillary pump,  $\bar{\Delta} = -\omega_i$ , there is an increase in the mechanical damping rate of the mechanical oscillator and consequently the cooling of the mechanical oscillator is provided. Moreover, the mechanical oscillator will be spring softened. In Fig. (4-a) and Fig. (4-b) we have plotted, respectively, the modified mechanical frequency and the modified optomechanical damping rate versus the normalized frequency  $\omega/\omega_i$  for two different values of the optomechanical coupling of the ancillary optical mode for experimentally feasible parameters of a typical optomechanical system [32, 43, 60]. With  $\omega = \omega_i$ , and in the resolved side-band limit ( $\omega_i \gg \kappa'$ ) the induced damping rate and the frequency shift of the oscillator are, respectively, given by

$$\gamma_{opt} = \frac{4|G|^2}{\kappa'}, \quad (\text{A10})$$

$$\omega_{opt} = -\frac{|G|^2}{2\omega_i}. \quad (\text{A11})$$

It is evident that  $\omega_m$  is negative for  $G > \sqrt{2}\omega_i$ . Under such a circumstance, the system can enter unstable region and the

Routh-Hurwitz stability conditions are violated [61].

Driving the system with an ancillary optical mode with a large damping rate ( $\kappa' \gg \kappa$  and  $\kappa' \gg \gamma_i$ ) tuned to the red side of the optical cavity [62, 63], as the consequence of the optomechanical interaction, the ancillary mode induces an optical damping to the mechanical oscillator. Working in the resolved-sideband regime and using a red detuned driving laser together with a weak optomechanical coupling the net effect of the ancillary optical mode is to renormalize the mechanical damping rate according to Eq. (A10). In this manner,  $\gamma_{opt}$  can be adjusted by the strength of the driving ancillary field so that the total damping rate of the mechanical oscillator,  $\gamma = \gamma_{opt} + \gamma_i$  becomes very large. Therefore, coupling a high-Q mechanical oscillator to an auxiliary cavity mode allows us to realize the RDR of the cavity optomechanics,  $\gamma \gg \kappa$  [49, 50] (see Figs. (1-b) and (1-c)). The wide separation between the time scales of dissipation mechanisms allows us to use the mechanical oscillator as an extra dissipative reservoir for the optical mode.

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